## SUPPLEMENTAL MATERIAL FOR A COMBINED FINITE ELEMENT AND FINITE VOLUME METHOD FOR LIQUID SIMULATION

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## Why we took different methods for FVM and FEM in pressure solve?

This supplemental material provides a bit of additional information for pressure solve proposed in our paper.

We originally imposed the second-order accurate Dirichlet boundary condition for the pressure solve using the method of Ando et al. [1] for both Finite Volume Method (FVM) and Finite Element Method (FEM). When we performed a water drop experiment with our discretization as shown in Fig.1, we faced the problem that artifacts occurred in the finite element method part and the boundary between FVM and FEM became visible like left side of Fig.2, even though the grids were not moving.

However, using Ibayashi et al. [2] instead of Ando et al. [1] as the second-order accurate Dirichlet boundary condition for FEM, the result was improved as shown in right side of Fig.2. Thus, we decided to use different second-order accuracy methods for FEM and FVM.

Note that the 3rd eigenvalue of the matrix in Eq.(36) of [2] needs correction. This point has been confirmed by the authors. Precisely, it is not  $\hat{b}_y$  but  $\hat{a}_y$ , and the equation is as follows:

$$M_R = \begin{pmatrix} \hat{a}_x / \hat{b}_x & \hat{a}_y / \hat{b}_x & 0\\ \hat{a}_y / \hat{b}_x & (\hat{a}_x^2 + 2\hat{a}_y^2) / (\hat{a}_x \hat{b}_x) & 0\\ 0 & 0 & (\hat{a}_x^2 + \hat{a}_y^2) / (\hat{a}_x \hat{b}_x) \end{pmatrix}.$$

## References

- [1] Ryoichi Ando and Christopher Batty. A practical octree liquid simulator with adaptive surface resolution. *ACM Trans. Graph.*, 39(4), July 2020.
- [2] H. Ibayashi, C. Wojtan, N. Thuerey, T. Igarashi, and R. Ando. Simulating liquids on dynamically warping grids. *IEEE Transactions on Visualization and Computer Graphics*, pages 1–1, 2018.



Figure 1: Setting for water drop test. FEM is used at the boundary of the red rectangle (not time-varying).



Figure 2: Rectangular artifact visible during water drop test. Left: Results using the method of Ando et al. Artifacts are visible. Right: Results using the method of Ibayashi et al.